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Abstract
The magnetoplasmon spectrum of a magnetically modulated two-dimensional electron gas (2DEG) is investigated. We derive the inter- and intra-Landau-band magnetoplasmon spectrum within the self-consistent field approach. The derivation is performed at zero temperature as well as at finite temperature. Results are presented for the inter- and intra-Landau-band magnetoplasmon spectrum as a function of the inverse magnetic field. Magnetic Weiss oscillations are found to occur in the magnetoplasmon spectrum as a result of magnetic modulation. Furthermore, our finite temperature theory facilitates the analysis of effects of temperature on the magnetoplasmon spectrum. The results are compared with those obtained for an electrically modulated 2DEG system. In addition, we derive and discuss the effects of simultaneous electric and magnetic modulations on the magnetoplasmon spectrum of the 2DEG when the modulations are in phase as well as when they are out of phase. Magnetic oscillations are affected by the relative phase of the two modulations and the position of the oscillations depends on the relative strength of the two modulations in the former case while we find complete suppression of Weiss oscillations for a particular relative strength of the modulations in the latter case.

1. The magnetoplasmon spectrum of magnetically modulated two-dimensional electron gas (MM2DEG)

In the past two decades, remarkable progress has been made in epitaxial crystal growth techniques, which have made possible the fabrication of novel semiconductor heterostructures. These modern microstructuring techniques can be used to laterally confine quasi-two-dimensional electron gas (2DEG) in e.g., a GaAs/AlGaAs heterostructure on a submicrometer scale. Furthermore, magnetic modulation of these systems can be realized by depositing an array of ferromagnetic strips on top of the heterostructure or by superconducting layers beneath the substrate. These magnetically modulated 2DEG systems realized in semiconductor heterostructures have attracted a lot of attention in the past and continue to do so today due to their novel properties [1–28]. Primarily, it is due to the introduction

of another length scale in the system, the period of modulation, as a result new phenomena arise due to commensurability of the period of modulation and the other characteristic length scale of the system, cyclotron diameter at the Fermi level. Resistivity measurements on these systems are found to exhibit commensurability oscillations (magnetic Weiss oscillations) since the magnetic modulation broadens the Landau levels into minibands whose width oscillates as a function of the magnetic field. These oscillations are periodic as a function of the inverse magnetic field with a larger period than that of the Shubnikov–de Hass (SdH) oscillations. The period of Weiss oscillations depends on both the modulation period and the square root of the number density of the MM2DEG, in contrast to the linear dependence on the number density of the SdH oscillations. Moreover, the amplitude of Weiss oscillations is weakly affected by temperature as compared to SdH oscillations.

We investigate the effects of magnetic modulation on the collective excitations (magnetoplasmons) of a 2DEG.

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Plasmons are a very general phenomenon and have been studied extensively in a wide variety of systems including ionized gases, simple metals and semiconductor 2DEG systems. In a 2DEG, these collective excitations are induced by the electron–electron interactions. These collective excitations, plasmons, are among the most important electronic properties of a system. In the presence of an external magnetic field, these collective excitations are known as magnetoplasmons. Magnetic oscillations of the plasmon frequency occur in a magnetic field. Single particle magneto-oscillatory phenomena such as the Shubnikov–de Haas and de Haas–van Alphen effects have provided very important probes of the electronic structure of solids. Their collective analog yields important insights into collective phenomena. For this reason we study the effects of magnetic modulation on the collective excitations (magnetoplasmons) in a 2DEG.

We present the magnetoplasmon spectrum of a MM2DEG in the presence of a perpendicular magnetic field using the self-consistent field approach. Both the inter- and intra-Landau-band magnetoplasmons are determined. Inter-Landau-band magnetoplasmons arise due to electronic transitions between different Landau bands whereas intra-Landau-band magnetoplasmons are a result of electronic transitions in a single Landau band. We evaluate the dynamic, nonlocal density–density response function to obtain these results. The collective excitations in a MM2DEG system have been investigated in the past [12, 16, 20, 21, 24]. All of these studies primarily investigate the inter-Landau-band magnetoplasmon mode. Reference [16] is an experimental study where the inter-Landau-band magnetoplasmons are investigated by infrared optical measurements whereas [12, 20, 21, 24] are theoretical studies. In [12] only the primary inter-Landau-band magnetoplasmon was considered. Far-infrared absorption of MM2DEG was theoretically investigated in [20] to explore plasma oscillations in the system. Similarly plasma oscillations were also theoretically investigated in [21, 24] taking into account primarily the inter-Landau-band magnetoplasmons. Since the existence of an intra-Landau-band magnetoplasmon is a result of the finite Landau bandwidth caused by modulation, the study of collective excitations and in particular modulation induced effects in this system requires taking into account the intra-Landau-band magnetoplasmons. To overcome this shortcoming in previous work mentioned above, we determine both the intra- and inter-Landau-band magnetoplasmon spectrum in this work. In addition, our finite temperature theory facilitates the analysis of effects of temperature on the Weiss and SdH oscillations in the magnetoplasmon spectrum of MM2DEG. Furthermore, we carry out a detailed comparison of phase and amplitude of magnetic Weiss oscillations and the electric Weiss oscillations in a 2DEG. We also present the effects of simultaneous electric and magnetic modulations on the magnetoplasmon spectrum of a 2DEG. To the best of our knowledge, the complete study of both the inter- and intra-Landau-band magnetoplasmon spectrum in this system and comparison of these with the results for the electrically modulated two-dimensional electron gas system (EM2DEG) [29–32] has not been carried out so far. To this end, we have undertaken the present study.

In section 2, we present the formulation of the problem. Section 3 contains the magnetoplasmon spectrum of a MM2DEG and comparison with an EM2DEG at zero temperature ($T = 0$) whereas in section 4 we discuss the temperature dependence of magnetoplasmon spectrum of a MM2DEG and its comparison with an EM2DEG including the asymptotic description. In the following two sections, magnetoplasmon spectrum in the presence of simultaneous electric and magnetic modulations is presented. We discuss the effects of in-phase modulations (electric and magnetic) in section 5 while out of phase is discussed in section 6. Concluding remarks are made in section 7.

2. Formulation

The system that we are considering is a 2DEG in the presence of a perpendicular magnetic field that is modulated weakly and periodically along one direction. We take the magnetic field B to be perpendicular to the x–y plane in which electrons with unmodulated areal density n_D, effective mass m^* and charge $-e$ are confined. We employ the Landau gauge and write the vector potential as $A = (0, Bx + (B_0/K) \sin Kx, 0)$, where K is $2\pi/a$, a is the period of modulation and B_0 is the magnetic modulation strength such that $B_0 \ll B$. The Hamiltonian in the Landau gauge is [5, 8, 26]

$$H_0 = \frac{1}{2m^*}\left[-\hbar^2 \frac{\partial^2}{\partial x^2} + \left(-i\hbar \frac{\partial}{\partial y} + eBx + (eB_0/K)\sin Kx \right)^2 \right]. \tag{1}$$

Since the Hamiltonian does not depend on the y coordinate, the unperturbed wavefunctions are plane waves in the y-direction. This allows us to write for the wavefunctions,

$$\phi_{nk_y}(\bar{x}) = \frac{1}{\sqrt{L_y}} e^{ik_y y} u_n(x), \tag{2}$$

with L_y being a normalization length in y-direction and $\bar{x}$ a 2D position vector in the x–y plane. The Hamiltonian in equation (1) can be expressed as

$$H_0 = -\frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial x^2} + \frac{1}{2} m^* \omega_c^2 (x - x_0)^2 + (\omega_0/K)$$
$$\times (p_y + eBx) \sin Kx + (m^* \omega_0^2/4K^2)[1 - \cos 2Kx], \tag{3}$$

where $\omega_c = \frac{eB}{m^*}$ is the cyclotron frequency, $\omega_0 = eB_0/m^*$ is the modulation frequency, $x_0 = -l^2 k_y = -\frac{\hbar k_y}{m^* \omega_c}$ is the coordinate of cyclotron orbit center, $l = \sqrt{\frac{\hbar}{m^* \omega_c}}$ is the magnetic length, and m^* is the effective mass. We can write the unmodulated eigenstates in the form $\phi_{nk_y}(\bar{x}) = \frac{1}{\sqrt{L_y}} e^{ik_y y} u_n(x; x_0)$, with $u_n(x; x_0) = (\sqrt{\pi} 2^n n! l)^{\frac{-1}{2}} \exp(-\frac{1}{2l^2}(x - x_0)^2) H_n(\frac{x-x_0}{l})$, where $u_n(x; x_0)$ is a normalized harmonic oscillator wavefunction centered at x_0 and $H_n(x)$ are Hermite polynomials with n the Landau level quantum number [33]. Since we are considering weak modulation $B_0 \ll B$, we can apply standard perturbation theory to determine the first order corrections to the unmodulated energy eigenvalues in the presence of modulation

$$\varepsilon(n, x_0) = (n + 1/2)\hbar\omega_c + G_n \cos\left(\frac{2\pi}{a} x_0\right) \tag{4}$$

where $G_n = \hbar\omega_0 \exp(-u/2)[L_n(u)/2 + L_{n-1}^1(u)], u = \frac{K^2 l^2}{2} = (\frac{2\pi}{a})^2 \frac{\hbar}{2m^*\omega_c}$ and $L_n(u), L_{n-1}^l(u)$ are Laguerre and associated Laguerre polynomials. This result has been obtained previously [5, 8]. The above equation shows that the formerly sharp Landau levels are now broadened into minibands by the modulation potential. Furthermore, the Landau bandwidth ($\sim |G_n|$) oscillates as a function of n, since $L_n(u)$ is an oscillatory function of its index [33].

Hereafter, we employ the Ehrenreich–Cohen self-consistent field (SCF) approach to determine the density–density response function [34]. Following the SCF approach, $\Pi_0(\bar{q}, \omega)$ is the density–density response function of the non-interacting electron system, given by

$$\Pi_0(\bar{q}, \omega) = \frac{1}{A} \sum_{n,n'} \sum_{k_y} C_{nn'}\left(\frac{\hbar\bar{q}^2}{2m^*\omega_c}\right)$$

$$\times \frac{f(\varepsilon(n', k_y - q_y)) - f(\varepsilon(n, k_y))}{\varepsilon(n', k_y - q_y) - \varepsilon(n, k_y) + \hbar\omega + i\hbar\eta}, \qquad (5)$$

where

$$C_{nn'}(x) = \frac{n_2!}{n_1!} e^{-x} x^{n_1 - n_2} [L_{n_2}^{n_1 - n_2}(x)]^2,$$

with $n_1 = \max(n, n')$, $n_2 = \min(n, n')$, $f(\varepsilon)$ is the Fermi–Dirac distribution function, $\bar{q}$ is the 2D wavenumber, A is the area of the system, $x = \frac{\hbar\bar{q}^2}{2m^*\omega_c}$ and $L_n^l(x)$ are associated Laguerre polynomials. The density–density response function of the interacting system can be expressed as

$$\Pi(\bar{q}, \omega) = \frac{\Pi_0(\bar{q}, \omega)}{1 - v_c(\bar{q})\Pi_0(\bar{q}, \omega)} \qquad (6)$$

with $v_c(\bar{q}) = \frac{2\pi e^2}{\kappa \bar{q}}$ is the 2D Coulomb potential, k is the background dielectric constant. Using the transformation $k_y \rightarrow -k_y$, realizing that $\varepsilon(n, k_y)$ is an even function of k_y, interchanging $n \leftrightarrow n'$ we can write the non-interacting density–density response function given by equation (5) as

$$\Pi_0(\bar{q}, \omega) = \frac{m^*\omega_c}{\pi\hbar a} \sum_{n,n'} C_{nn'}\left(\frac{\hbar\bar{q}^2}{2m^*\omega_c}\right)$$

$$\times \int_0^a dx_0 \left[f(\varepsilon(n, x_0 + x_0')) - f(\varepsilon(n', x_0))\right]$$

$$\times [\varepsilon(n, x_0 + x_0') - \varepsilon(n', x_0) + \hbar\omega + i\hbar\eta]^{-1}. \qquad (7)$$

In writing the above equation we converted the k_y-sum into an integral over x_0 and $x_0' = -\frac{\hbar q_y}{m^*\omega_c}$.

3. Magnetoplasmons modes of MM2DEG and comparison with EM2DEG at zero temperature

The plasma modes are obtained from the roots of the longitudinal dispersion relation from equations (6) and (7)

$$1 - v_c(\bar{q}) \, \text{Re} \, \Pi_0(\bar{q}, \omega) = 0 \qquad (8)$$

along with the condition $\text{Im} \, \Pi_0(\bar{q}, \omega) = 0$ to ensure long-lived excitations. The roots of equation (8) give the plasma modes of a MM2DEG as

$$1 = \frac{2\pi e^2}{\kappa \bar{q}} \frac{2}{\pi l^2} \sum_{n,n'} \sum C_{nn'}(x)\{I_1(n, n', x_0'; \omega)$$

$$+ I_1(n, n', x_0'; -\omega)\}, \qquad (9)$$

with

$$I_1(n, n', x_0'; \omega) = P \int_0^a dx_0$$

$$\times \frac{f(\varepsilon(n, x_0))}{\varepsilon(n', x_0 + x_0') - \varepsilon(n, x_0 + x_0') + \hbar\omega}, \qquad (10)$$

and P is the principal value. From here on we will only show the dependence of I_1 on ω and suppress the rest such that $I_1(n, n', x_0'; \omega) \rightarrow I_1(\omega)$.

SdH and Weiss oscillations are found to occur in the magnetoconductivity of both electric and magnetically modulated 2DEG. These transport measurements can be explained without taking into account electron–electron interactions. In order to investigate collective excitations of the system such as magnetoplasmons it is essential to consider electron–electron interactions. Magnetoplasmons arise due to the coherent motion of electrons as a result of electron–electron interactions. Two types of magnetoplasmons can be identified: those arising from electronic transitions involving different Landau bands (inter-Landau-band plasmons) and those within a single Landau band (intra-Landau-band plasmons). Inter-Landau-band plasmons involve the local 1D magnetoplasma mode and the Bernstein-like plasma resonances [35, 36], all of which involve excitation energies greater than the Landau-band separation ($\sim \hbar\omega_c$). On the other hand, intra-Landau-band magnetoplasmons resonate at energies comparable to the bandwidths, and the existence of this new class of modes is due to finite width of the Landau levels. In a MM2DEG considered here, the Landau bandwidth ($\sim |G_n|$) oscillates as a function of the band index n, since $L_n(u)$, $L_n^l(u)$ are oscillatory functions of the index n. Such oscillating bandwidths affect the plasmon spectrum of the intra-Landau-band type, when Landau-band separation is larger than the bandwidth as is the case considered here, resulting in magnetic Weiss oscillations similar to the electric Weiss oscillations found in the electrically modulated system. These oscillations are accompanied by SdH type of oscillatory behavior [29, 37]. Both these oscillations are periodic as a function of inverse magnetic field $(1/B)$ but occur with different periods and amplitudes. As we show below, Weiss oscillations in the magnetoplasmon spectrum of a MM2DEG differ in phase and amplitude with those of an EM2DEG [29].

We now examine the inter-Landau-band transitions. These transitions occur between different Landau bands so we consider $n \neq n'$ in equation (10) which yields

$$I_1(\omega) = \frac{f(\varepsilon(n))}{(\hbar\omega - \Delta)},$$

where $\Delta = (\varepsilon(n) - \varepsilon(n'))$ with $\varepsilon(n) = (n + \frac{1}{2})\hbar\omega_c$, which permits us to write the following term in equation (9) as

$$(I_1(\omega) + I_1(-\omega)) = 2\frac{\Delta f(\varepsilon(n))}{(\hbar\omega)^2 - (\Delta)^2}. \qquad (11)$$

Next, we consider the coefficient $C_{nn'}(x)$ in equation (9) and expand it to lowest order in its argument (low wavenumber expansion). In this case, we are only considering the $n' = n \pm 1$ terms. The inter-Landau-band plasmon modes under consideration arise from neighboring Landau bands.

Hence for $n' = n + 1$ and $x \ll 1$, using the following associated Laguerre polynomial expansion [38] $L_n^l(x) = \sum_{m=0}^{n}(-1)^m \frac{(n+l)!}{(l+m)!(n-m)!} \frac{x^m}{m!}$ for $l > 0$ and retaining the first term in the expansion for $x \ll 1$, $C_{nn'}(x)$ reduces to

$$C_{n,n+1}(x) \to (n+1)x, \qquad (12)$$

and for $n' = n - 1$ and $x \ll 1$, it reduces to

$$C_{n,n-1}(x) \to nx. \qquad (13)$$

Substitution of equations (11)–(13) into (9) and replacing $x = \frac{\hbar \bar{q}^2}{2m^* \omega_c}$ yields

$$1 = \frac{2\pi e^2}{km^*} \bar{q} \frac{1}{\omega^2 - \omega_c^2} \left(\frac{m\omega_c}{\pi \hbar} \sum_n f(\varepsilon(n)) \right).$$

The term in parenthesis is easily recognized as the unmodulated particle density $n_D = \frac{m\omega_c}{\pi \hbar} \sum_n f(\varepsilon(n))$, where the summation is over all occupied Landau bands. Defining the plasma frequency through $\omega_{p,2D}^2 = \frac{2\pi n_D e^2}{km} \bar{q}$, we obtain the inter-Landau-band plasmon dispersion relation $1 = \frac{\omega_{p,2D}^2}{\omega^2 - \omega_c^2}$ or

$$\omega^2 = \omega_c^2 + \omega_{p,2D}^2, \qquad (14)$$

here ω is the well-known local 2D principal plasma frequency. Since we are interested in the modulation induced effects on the magnetoplasmons in this system, our focus will be on the intra-Landau-band magnetoplasmons as the dispersion relation for inter-Landau-band magnetoplasmons have been discussed and results displayed in [12, 29].

For the intra-Landau-band excitations spectrum, we need to consider transitions within a Landau miniband, i.e. $n = n'$, $\varepsilon(n') - \varepsilon(n) = 0$ and $C_{nn'}(x) \to 1$. An analytical expression of the intra-Landau-band plasmon energy $\hbar \tilde{\omega}$ can be obtained

$$\hbar^2 \tilde{\omega}^2 = \frac{16e^2}{k\bar{q}\pi} \frac{m^* \omega_c}{\hbar a} \sin^2 \left(\frac{\pi}{a}(x_0') \right) A_n, \qquad (15)$$

where

$$A_n = \sum_n G_n \int_0^{a/2} dx_0 f(\varepsilon(n, x_0)) \cos(Kx_0).$$

At zero temperature ($T = 0$),

$$\hbar^2 \tilde{\omega}^2 = \frac{8e^2}{k\bar{q}} \frac{m^* \omega_c}{\pi \hbar} \sin^2 \left(\frac{\pi}{a}(x_0') \right) \sum_n |G_n|$$

$$\times \sqrt{1 - \Delta_n^2} \theta(1 - \Delta_n), \qquad (16)$$

with $\Delta_n = |\frac{\varepsilon_F - \varepsilon(n)}{G_n}|$, $\theta(x)$ the Heaviside unit step function. If we replace the magnetic modulation term (G_n) by the electric one, equation (16) has the same structure as equation (8) of [29] that pertains to electric modulation. The above expression for $\hbar \tilde{\omega}$ has been obtained under the condition $\hbar \omega \gg |\varepsilon(n, x_0 + x_0') - \varepsilon(n, x_0)|$ as $x_0' \to 0$ which leads to a relation between the energy and the Landau level broadening $\hbar \omega \gg |2G_n \sin(\frac{\pi}{a}x_0') \sin[(\frac{2\pi}{a})(x_0 + \frac{x_0'}{2})]|$. This ensures that Im $\Pi_0(\bar{q}, \omega) = 0$ and the intra-Landau-band
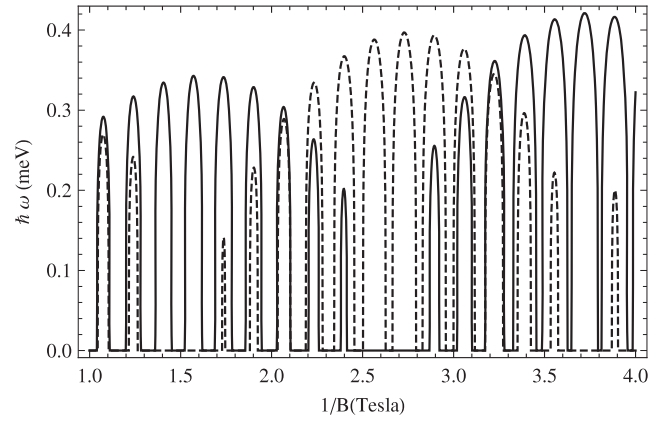

Figure 1. Intra-Landau-band plasma energy as a function of inverse magnetic field at zero temperature: the solid line shows the magnetic modulations case while the dashed line shows the electric modulations case (final result of [29]). Here the strength of electric modulations is 0.6 and 0.07 meV for the magnetic modulation.

magnetoplasmons are undamped. For a given G_n, this can be achieved with a small but nonzero q_y (recall that $x_0' = -\frac{\hbar q_y}{m^* \omega_c}$).

In general, the inter- and intra-Landau-band modes are coupled for arbitrary magnetic field strengths. The general dispersion relation is

$$1 = \frac{\omega_{p,2D}^2}{\omega^2 - \omega_c^2} + \frac{\tilde{\omega}^2}{\omega^2}.$$

This equation yields two modes which are given by

$$\omega_{\pm}^2 = \frac{1}{2}(\omega_c^2 + \omega_{p,2D}^2 + \tilde{\omega}^2) \pm \frac{1}{2}\{(\omega_c^2 + \omega_{p,2D}^2 + \tilde{\omega}^2 + 2\omega_c \tilde{\omega})(\omega_c^2 + \omega_{p,2D}^2 + \tilde{\omega}^2 - 2\omega_c \tilde{\omega})\}^{1/2}$$

which reduces to

$$\omega_+^2 = \omega_c^2 + \omega_{p,2D}^2,$$

and

$$\omega_-^2 = \tilde{\omega}^2,$$

with corrections of order $\tilde{\omega}^2/\omega_c^2$ and $\tilde{\omega}^2/\omega_{p,2D}^2$. So long as $|G_n| < \hbar \omega_c$, mixing of the inter- and intra-band modes is small. Only the intra-Landau-band mode ($\hbar \tilde{\omega}$) will be excited in the frequency regime $\hbar \omega_c > \hbar \omega \sim |G_n|$.

The intra-Landau-band plasma energy given by equation (11) is shown graphically in figure 1 as a function of $1/B$. The parameters used are [5, 7, 8, 26, 29–31]: $m^* = 0.07m_e$, $k = 12$, $n_D = 3.16 \times 10^{15}$ m^{-2}, $a = 380$ nm. We also take $q_x = 0$ and $q_y = 0.01k_F$, with $k_F = (2\pi n_D)^{1/2}$ being the Fermi wavenumber of the unmodulated 2DEG in the absence of magnetic field. In numerical evaluation we have taken the sum over thirty Landau levels which ensures convergence of numerical results. Numerical evaluation of the dispersion relation was performed in Mathematica. In figure 1, modulation induced oscillations in the intra-Landau-band mode are apparent, superimposed on SdH type oscillations. These oscillations are periodic as a function of inverse magnetic field ($1/B$). To gain further insight into the results

presented in figure 1, we consider equation (16). In the regime $\hbar\omega_c > |G_n|$, the unit step function vanishes for all but the highest occupied Landau band, corresponding, say, to the band index N. The sum over n is trivial and plasma energy is given as $\hbar\tilde{\omega} = |G_N|^{1/2}(1 - \Delta_N^2)^{1/4}\theta(1 - \Delta_N)$. The analytic structure primarily responsible for the SdH type of oscillations is the function $\theta(1 - \Delta_N)$, which jumps periodically from zero (when the Fermi level is above the highest occupied Landau band) to unity (when the Fermi level is contained with in the highest occupied Landau band). On the other hand, the periodic modulation of the amplitude of the SdH type oscillations is due to the oscillatory nature of the factor $|G_N|^{1/2}$, which has been shown to exhibit commensurability oscillations [6, 19, 21, 31]. In the same figure, we also show the intra-Landau-band plasma energy for the electrically modulated 2DEG [29]. The magnetoplasmons spectrum of the MM2DEG at a specific Landau level is minimum when the corresponding spectrum for the EM2DEG is maximum, this confirms that magnetic oscillations in the two system are out of phase. To have comparable results for the two systems, at zero temperature, the strength of magnetic modulation potential, $\hbar\omega_0$, has to be ~ 8.5 times smaller than that of the electric modulation potential, V_0. This can be understood if we realize that this constraint arises due to the step function $\theta(1 - \Delta_N)$ appearing in the dispersion relation at zero temperature. Therefore, comparable results for the two systems when they are subjected to modulation of equal strength requires that we carry out a finite temperature calculation to avoid the constraint imposed by the step function at zero temperature.

4. The temperature dependent magnetoplasmons mode of MM2DEG and a comparison with EM2DEG

For the finite temperature calculation of the intra-landau-band plasma energy, we invoke the condition of weak modulation and perform the following expansion in equation (15)

$$f(\varepsilon(n, x_0)) \simeq f(\varepsilon(n)) + G_n f'(\varepsilon(n)) \cos(Kx_0), \qquad (17)$$

where $f'(x) = \frac{d}{dx}f(x)$ is the derivative of the Fermi–Dirac distribution function. With the substitution of this expansion in equation (15), the integral over x_0 contains two terms. The integral over x_0 of the first term vanishes and the integral of the second term yields the intra-Landau-band dispersion relation

$$\hbar^2\tilde{\omega}^2 = \frac{4e^2 m^* \omega_c}{k\bar{q}\hbar\pi} \sin^2\left[\frac{\pi}{a}(x_0')\right] B_n, \qquad (18)$$

where

$$B_n = \sum_n G_n^2 [-f'(\varepsilon(n))]. $$

To facilitate comparison of the above dispersion relation with the results obtained for an electrically modulated 2DEG we will determine the asymptotic expressions of the intra-Landau-band magnetoplasmon spectrum, where analytic results in terms of elementary functions can be obtained. Moreover, these asymptotic expressions will allow us to identify terms responsible for SdH and Weiss oscillations and how they are affected by temperature.

The asymptotic expression can be obtained by using the following asymptotic expression for the Laguerre polynomials [5, 7, 8, 26]

$$\exp^{-u/2} L_n(u) \to \frac{1}{\sqrt{\pi\sqrt{nu}}} \cos\left(2\sqrt{nu} - \frac{\pi}{4}\right). \qquad (19)$$

Note that the above asymptotic expression for $L_n(u)$ is valid for $n \gg 1$, at low magnetic fields when many Landau Levels are filled. We now take the continuum limit:

$$n --> \frac{\varepsilon(n)}{\hbar\omega_c}, \qquad \sum_{n=0}^{\infty} --> \int_0^{\infty} \frac{d\varepsilon}{\hbar\omega_c}. \qquad (20)$$

In the asymptotic limit, B_n that appears in equation (18) can be written as

$$B_n = \frac{\hbar^2\omega_0^2}{\pi\hbar\omega_c} \sqrt{\frac{\hbar\omega_c}{u}} \left(\frac{aK_F}{2\pi}\right)^2$$
$$\times \int_0^{\infty} \frac{d\varepsilon}{\sqrt{\varepsilon}} \frac{\beta g(\varepsilon)}{[g(\varepsilon) + 1]^2} \sin^2\left(2\sqrt{nu} - \frac{\pi}{4}\right), \qquad (21)$$

where $g(\varepsilon) = \exp[\beta(\varepsilon - \varepsilon_F)]$, $\beta = \frac{1}{K_B T}$.

Now assuming that temperature is low such that $\beta^{-1} \ll \varepsilon_F$ and replacing $\varepsilon = \varepsilon_F + s\beta^{-1}$, we can express the above integral as

$$B_n = \frac{\hbar^2\omega_0^2}{\pi\sqrt{u\hbar\omega_c\varepsilon_F}} \left(\frac{aK_F}{2\pi}\right)^2 \int_0^{\infty} ds \frac{e^s}{[(e^s + 1)]^2}$$
$$\times \sin^2\left(2\sqrt{\frac{u\varepsilon_F}{\hbar\omega_c}} - \frac{\pi}{4} + \sqrt{\frac{u}{\hbar\omega_c\varepsilon_F}} s\beta^{-1}\right)$$

with the result

$$B_n = \frac{\hbar^2\omega_0^2}{2\pi\sqrt{u\hbar\omega_c\varepsilon_F}} \left(\frac{aK_F}{2\pi}\right)^2$$
$$\times \left[1 - A\left(\frac{T}{T_a}\right) + 2A\left(\frac{T}{T_a}\right)\sin^2\left[2\sqrt{\frac{u\varepsilon_F}{\hbar\omega_c}} - \frac{\pi}{4}\right]\right], \quad (22)$$

where T_a is the characteristic damping temperature of Weiss oscillation given by $k_B T_a = \frac{\hbar\omega_c aK_F}{4\pi^2}$, $\frac{T}{T_a} = \frac{4\pi^2 k_B T}{\hbar\omega_c aK_F}$ and $A(x) = \frac{x}{\sinh(x)} -^{(x-->\infty)} -> = 2xe^{-x}$.

From equation (18), the asymptotic expression for intra-Landau-band plasmon spectrum is obtained

$$\hbar^2\tilde{\omega}^2 = \frac{4\hbar^2\omega_0^2 e^2 m^*\omega_c}{k\bar{q}\hbar 2\pi^2 \sqrt{u\hbar\omega_c\varepsilon_F}} \left(\frac{aK_F}{2\pi}\right)^2 \sin^2\left[\frac{\pi}{a}(x_0')\right]$$
$$\times \left[1 - A\left(\frac{T}{T_a}\right) + 2A\left(\frac{T}{T_a}\right)\sin^2\left(2\sqrt{\frac{u\varepsilon_F}{\hbar\omega_c}} - \frac{\pi}{4}\right)\right]. \tag{23}$$

The above expression is not able to account for the SdH type of oscillations in the magnetoplasmon spectrum. These oscillations can be accounted for by expressing the density of states (in the absence of disorder) [5] as

$$D(\varepsilon) = \frac{m^*}{\pi\hbar}\left(1 - 2\cos\left[\frac{2\pi\varepsilon}{\hbar\omega_c}\right]\right) \tag{24}$$

and inserting the continuum approximation as $\sum_{n=0}^{\infty} --> 2\pi l^2 \int_0^{\infty} D(\varepsilon) d\varepsilon$, this yields the asymptotic expression for
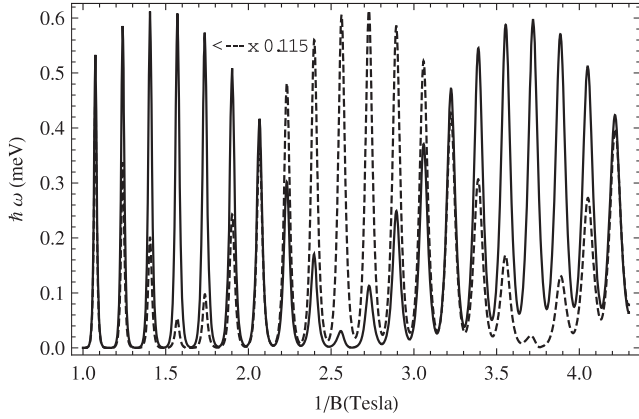
Figure 2. Intra-Landau-band plasma energy as a function of inverse magnetic field at *T* 0.4 K. Here the strength of electric and magnetic modulation is the same as 1 meV. Solid and dashed lines represent magnetically and electrically modulated 2DEG, respectively.
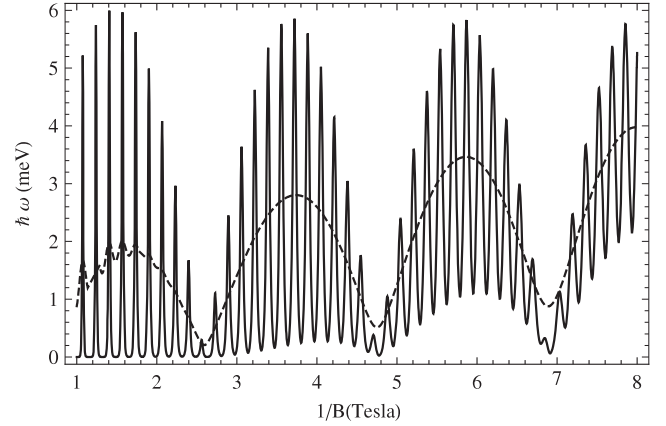

Figure 3. Intra-Landau-band plasma energy of a MM 2DEG as a function of the inverse magnetic field at $T = 0.3$ K (solid line) and $T = 3$ K (dashed line)

the intra-Landau-band magnetoplasmon dispersion relation for MM2DEG

$$\hbar^2 \tilde{\omega}^2 = \frac{4\hbar^2 \omega_0^2 e^2 m^* \omega_c}{k\bar{q}\hbar 2\pi^2 \sqrt{u\hbar\omega_c \varepsilon_F}} \left(\frac{aK_F}{2\pi}\right)^2 \sin^2\left[\frac{\pi}{a}(x_0')\right]$$
$$\times \left\{ \left[1 - A\left(\frac{T}{T_a}\right) + 2A\left(\frac{T}{T_a}\right)\sin^2\left(2\sqrt{\frac{u\varepsilon_F}{\hbar\omega_c}} - \frac{\pi}{4}\right)\right] \right.$$
$$\left. - 4A\left(\frac{T}{T_s}\right)\cos\left[\frac{2\pi\varepsilon_F}{\hbar\omega_c}\right]\sin^2\left[2\sqrt{\frac{u\varepsilon_F}{\hbar\omega_c}} - \frac{\pi}{4}\right]\right\}, \quad (25)$$

where $\frac{T}{T_s} = \frac{2\pi^2 k_B T}{\hbar\omega_c}$, T_s defines the characteristic damping temperature of the SdH oscillations in the magnetoplasmon spectrum of MM2DEG.

Following the same approach as discussed above for MM2DEG, we can obtain the intra-Landau-band magnetoplasmon spectrum for EM2DEG

$$\hbar^2 \tilde{\omega}^2 = \frac{4e^2 m^* \omega_c}{\hbar k\bar{q}\pi} \sin^2\left[\frac{\pi}{a}(x_0')\right] \times B_n, \quad (26)$$

where $B_n = \sum F_n^2 \times [-f'(\varepsilon(n))]$, and $F_n = V_0 e^{-u/2} L_n(u)$ is the modulation width of the EM2DEG with V_0 the amplitude of electric modulation. The corresponding asymptotic result for EM2DEG is

$$\hbar^2 \tilde{\omega}^2 = \frac{4V_0^2 e^2 m^* \omega_c}{k\bar{q}2\pi^2\hbar\sqrt{u\hbar\omega_c \varepsilon_F}} \sin^2\left(\frac{\pi}{a}(x_0')\right)\left\{\left[1 - A\left(\frac{T}{T_a}\right)\right.\right.$$
$$\left. + 2A\left(\frac{T}{T_a}\right)\cos^2\left(2\sqrt{\frac{u\varepsilon_F}{\hbar\omega_c}} - \frac{\pi}{4}\right)\right] - 4A\left(\frac{T}{T_s}\right)$$
$$\left. \times \cos\left[\frac{2\pi\varepsilon_F}{\hbar\omega_c}\right]\cos^2\left[2\sqrt{\frac{u\varepsilon_F}{\hbar\omega_c}} - \frac{\pi}{4}\right]\right\}. \quad (27)$$

The intra-landau-band plasmon dispersion relations obtained for the MM2DEG and the EM2DEG systems given by equations (25) and (27) allow us to identify the terms responsible for Weiss and SdH oscillations. Moreover, the characteristic damping temperatures appearing in these expressions carry the effects of temperature on these oscillations. Comparing equations (25) and (27), the following differences can be highlighted:

(i) Amplitude of the oscillations (Weiss and SdH) are larger by the factor $\frac{aK_F}{2\pi}$ in MM2DEG compared to those of EM2DEG.

(ii) The factor $\sin^2[2\sqrt{\frac{u\varepsilon_F}{\hbar\omega_c}} - \frac{\pi}{4}]$ that appears in equation (25) for MM2DEG and the corresponding factor $\cos^2[2\sqrt{\frac{u\varepsilon_F}{\hbar\omega_c}} - \frac{\pi}{4}]$ in equation (27) for EM2DEG results in a $\pi/2$ phase difference in the oscillations in the magnetoplasmon spectrum of the two systems

Since equations (25) and (27) are the key results of this work, we show the intra-Landau-band magnetoplasmon energy for both the magnetically and electrically modulated systems as a function of inverse magnetic field in figure 2. The results presented are for two modulations of equal strength, which is taken to be $V_0 = \hbar\omega_0 = 1$ meV. The temperature is 0.4 K. The rest of the parameters are the same as the case for the zero temperature results presented in figure 1. We observe the modulation induced effects in the intra-Landau-band mode, Weiss oscillations modulating the SdH oscillations in the magnetoplasmon spectrum. From the figure, we see that the amplitude of Weiss oscillations in the magnetically modulated system is greater by a factor of ~8.5 compared to the electrically modulated system. This can be seen from equations (25) and (27) where the difference in the amplitudes is the factor $\frac{aK_F}{2\pi}$ and it is ~8.5 for the parameters that we have used. Therefore, in the magnetically modulated system the amplitude is larger by this factor compared to the electrically modulated system. We also observe that Weiss oscillations in the MM2DEG are out of phase by $\pi/2$ compared to those in EM2DEG. To see the effects of temperature on Weiss and SdH oscillations in a MM2DEG, we plot the intra-Landau-band plasmon energy for a MM2DEG as a function of inverse magnetic field at two different temperatures in figure 3. The modulation strength is 1 meV. The results are shown at the following two temperatures: 0.3 and 3 K. The SdH oscillations are completely damped at 3 K whereas Weiss oscillations persist at this temperature. Equations (25) and (27) also allow us to determine the temperature scales for damping of Weiss and SdH oscillations in the magnetoplasmon spectrum. For
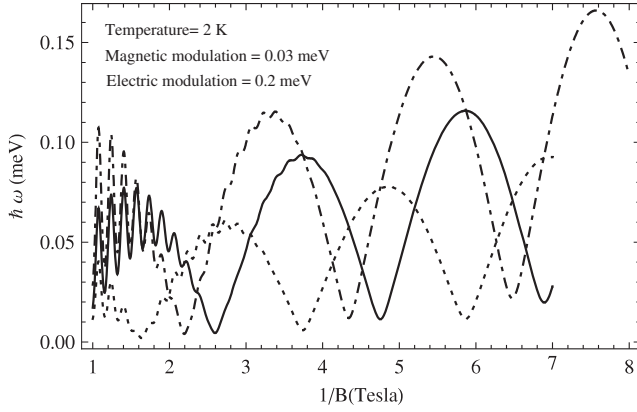
Figure 4. Intra-Landau-band plasma energy as a function of inverse magnetic field in the presence of a magnetic and an electric in-phase modulation (dot–dashed line). Magnetic modulation alone (solid line), electric modulation alone (dashed line).
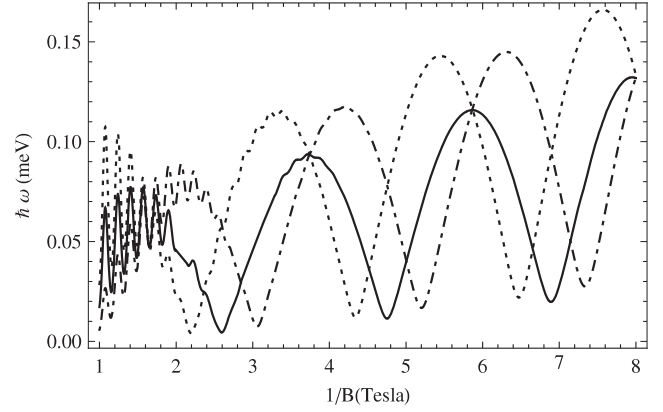

Figure 5. Intra-Landau-band plasma energy as a function of inverse magnetic field in the presence of a magnetic and an electric in-phase modulation at $T = 2$ K. Magnetic modulation is fixed at 0.03 meV and electric modulation is varied: 0 meV (solid), +0.2 meV (dotted) and −0.2 meV (dot–dashed line.)

a MM2DEG, from equation (25), $\frac{T_a}{T_s} = \frac{aK_F}{2} \gg 1$; e.g., $n_D = 3.16 \times 10^{15}$ m^{-2} and $a = 380$ nm, we have $\frac{T_a}{T_s} = 27$ for the experimentally relevant parameters considered here. Hence, SdH oscillations are completely damped at a much lower temperature compared to Weiss oscillations. These results are consistent with and complement those obtained from electron transport studies of a magnetically modulated 2DEG [5–8, 19–21, 26, 27].

5. The magnetoplasmon spectrum with periodic electric and magnetic modulation: in phase

In this section, we calculate the magnetoplasmon spectrum when electric and magnetic modulations are in phase. We take the magnetic modulation to have the same phase as given in the previous section with the in-phase electric modulation. The energy eigenvalues are [5, 26]

$$\varepsilon(n, x_0) = (n + 1/2)\hbar\omega_c + (G_n + F_n)\cos(Kx_0) \quad (28)$$

and the bandwidth can be written as

$$\Delta(\text{in phase}) = \frac{2\hbar\omega_0 ak_F\sqrt{1+\delta^2}}{2\pi\sqrt{\pi\sqrt{nu}}}$$
$$\times \sin\left(2\sqrt{nu} - \frac{\pi}{4} + \Phi\right), \quad (29)$$

where the ratio between the two modulation strengths $\delta = \frac{2\pi V_0}{\hbar\omega_0 ak_F} = \tan(\Phi)$. The flat band condition from the above equation is $2\sqrt{nu} - \frac{\pi}{4} + \Phi = i\pi$ where i is an integer. This condition can also be expressed as $\frac{\sqrt{2n}}{a}l = i + \frac{1}{4} - \frac{\Phi}{\pi}$, where $n = n_F = \frac{\varepsilon_F}{\hbar\omega_c} - \frac{1}{2}$ is the highest Fermi integer. We see that the flat band condition in this case depends on the relative strength of the two modulations.

Following the same approach as discussed in the previous section for the MM2DEG, we can obtain the intra-Landau-band magnetoplasmon spectrum in the presence of in-phase modulations as

$$\hbar^2\tilde{\omega}^2 = \frac{4e^2m^*\omega_c}{k\bar{q}\pi\hbar}\sin^2\left[\frac{\pi}{a}(x_0')\right]I_n, \quad (30)$$

where

$$I_n = \sum_n (G_n + F_n)^2[-f'(\varepsilon(n))].$$

In figure (4) we show the in-phase magnetoplasmon spectrum (the magnetic and the electric modulations are in phase) $\hbar\tilde{\omega}$ given by equation (30) as a function of the inverse magnetic field for temperature $T = 0.3$ K, electron density $n_e = 3 \times 10^{11}$ cm^{-2}, the period of modulation $a = 380$ nm. The strength of the electric modulation $V_0 = 0.2$ meV whereas $B_0 = 0.02$ T, which corresponds to $\hbar\omega_0 = 0.03$ meV. In the same figure we have also shown the magnetoplasmon spectrum when either the magnetic or electric modulation alone is present. The $\frac{\pi}{2}$ phase difference in the bandwidths results in the same phase difference appearing in the magnetoplasmon spectrum for electric and magnetic modulations as can be seen in the figure. To better understand the effects of in-phase modulations on the magnetoplasmon spectrum we consider the asymptotic expression of the magnetoplasmon spectrum given by equation (30). The asymptotic expression is given by

$$\hbar^2\tilde{\omega}^2 = \frac{2V_0^2 e^2 m^*\omega_c\delta^{-2}}{k\bar{q}2\pi^2\hbar\sqrt{u\hbar\omega_c\varepsilon_F}}\sin^2\left(\frac{\pi}{a}(x_0')\right)(1+\delta^2)$$
$$\times \left\{\left[1 - A\left(\frac{T}{T_a}\right) + \left(2A\left(\frac{T}{T_a}\right) - 4A\left(\frac{T}{T_s}\right)\right.\right.\right.$$
$$\times \left.\cos\left[\frac{2\pi\varepsilon_F}{\hbar\omega_c}\right]\right)\sin^2\left[2\sqrt{\frac{u\varepsilon_F}{\hbar\omega_c}} - \frac{\pi}{4} + \Phi\right]\right\}. \quad (31)$$

From the asymptotic expression given by equation (31), we observe that in the presence of in-phase electric and magnetic modulation the magnetoplasmon energy acquires a dependence on the phase factor Φ and δ which depend on the relative modulation strengths. The shift in the Weiss oscillations when in-phase electric and magnetic modulations are present can be seen in figure 5. How the Weiss oscillations are affected as Φ as well as the magnetic field is varied can be seen in figure 5. The results shown are for a fixed magnetic modulation of strength $\hbar\omega_0 = 0.03$ meV and the electric modulation is varied. The change in V_0 results in a corresponding change in both δ and Φ. From figure 5, we observe that the position of

the extrema in the magnetoplasmon spectrum as a function of the inverse magnetic field depend on the relative strength of the modulations.

The effects of electric and magnetic modulations that are out of phase on the magnetoplasmon spectrum can be better appreciated if we consider the asymptotic expression in this case. This is taken up in section 6.

6. Magnetoplasmons with periodic electric and magnetic modulation: out of phase

In this section, we calculate the magnetoplasmon spectrum when electric and magnetic modulations are out of phase by $\pi/2$. We consider magnetic modulation out of phase with the electric one: we take the electric modulation to have the same phase as given in the previous section with the $\pi/2$ phase difference incorporated in the magnetic field. The energy eigenvalues are [5, 26]

$$\varepsilon(n, x_0) = (n+1/2)\hbar\omega_c + \sin(Kx_0)G_n + F_n\cos(Kx_0), \quad (32)$$

and the bandwidth is

$$\Delta(\text{out of phase}) = \frac{2\hbar\omega_0 a k_F}{2\pi\sqrt{\pi}\sqrt{nu}}$$

$$\times \sqrt{\delta^2 + (1-\delta^2)\sin\left(2\sqrt{nu} - \frac{\pi}{4}\right)}. \quad (33)$$

The term responsible for Weiss oscillations is the $\sin(2\sqrt{nu} - \frac{\pi}{4})$ term under the square root which can be readily seen by considering the large n limit of the bandwidth. Therefore for $\delta = \pm1$ Weiss oscillations are no longer present in the bandwidth.

Following the same approach discussed in the previous section for MM2DEG, we can obtain the intra-Landau-band magnetoplasmon spectrum in the presence of out-of-phase modulations as

$$\hbar^2\tilde{\omega}^2 = \frac{4e^2m^*\omega_c}{k\bar{q}\pi\hbar}\sin^2\left[\frac{\pi}{a}(x_0')\right]O_n, \quad (34)$$

where

$$O_n = \sum_n (G_n^2 + F_n^2)[-f'(\varepsilon(n))].$$

The asymptotic expression in the presence of both electric and magnetic modulations that are out of phase is obtained by substituting the asymptotic expressions for the Laguerre polynomials and converting the sum into integration with the result

$$\hbar^2\tilde{\omega}^2 = \frac{2V_0^2 e^2 m^*\omega_c\delta^{-2}}{k\bar{q}2\pi^2\hbar\sqrt{u\hbar\omega_c\varepsilon_F}}\sin^2\left(\frac{\pi}{a}(x_0')\right)\Big\{2\delta^2 + (1-\delta^2)$$

$$\times\left[1 - A\left(\frac{T}{T_a}\right) + 2A\left(\frac{T}{T_a}\right)\sin^2\left[2\sqrt{\frac{u\varepsilon_F}{\hbar\omega_c}} - \frac{\pi}{4}\right]\right]$$

$$- 4A\left(\frac{T}{T_s}\right)\cos\left[\frac{2\pi\varepsilon_F}{\hbar\omega_c}\right]\left[\delta^2 - (\delta^2 - 1)\right.$$

$$\times\left.\sin^2\left[2\sqrt{\frac{u\varepsilon_F}{\hbar\omega_c}} - \frac{\pi}{4}\right]\right]\Big\}. \quad (35)$$
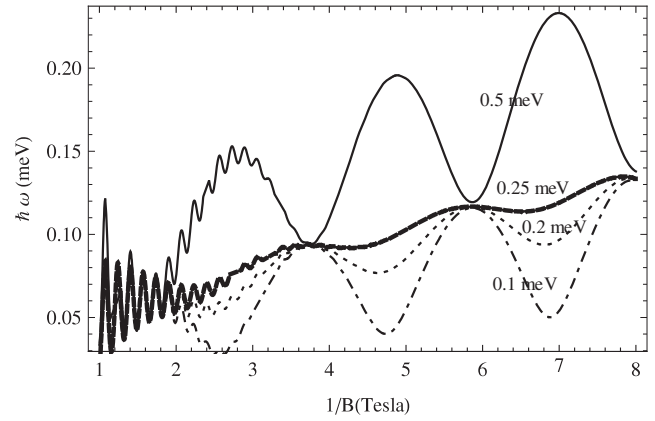

Figure 6. Intra-Landau-band plasma energy as a function of inverse magnetic field in the presence of a magnetic and an electric out-of-phase modulation at $T = 2$ K. Magnetic modulation is fixed at 0.03 meV and electric modulation is varied.

From the expression of the out-of-phase bandwidth given by equation (33) we find that Weiss oscillations in the bandwidth are absent for relative modulation strength $\delta = \pm1$, the same is reflected in the magnetoplasmon spectrum as the term responsible for Weiss oscillations ($\sin^2[2\sqrt{\frac{u\varepsilon_F}{\hbar\omega_c}} - \frac{\pi}{4}]$) vanishes for $\delta = \pm1$ as can be seen from the above equation. Therefore the magnetoplasmon spectrum does not exhibit Weiss oscillations when the relative modulation strength $\delta = \pm1$. The spectrum as a function of magnetic field when the electric and magnetic modulations are out of phase is shown in figure 6. The results shown are for a fixed magnetic modulation of strength $\hbar\omega_0 = 0.03$ meV and the electric modulation V_0 is allowed to vary between positive and negative values. The other parameters are the same as in figures 4 and 5. As V_0 is varied there is a corresponding change in δ. We find that the positions of the extrema of the spectrum as a function of the inverse magnetic field do not change as δ is varied since the phase factor Φ does not appear in the expression of the magnetoplasmon spectrum when the two modulations are out of phase. It is also observed in figure 6 that there is a $\frac{\pi}{2}$ phase difference between the curves for $\delta \geqslant 1$ and $\delta < 1$. The same behavior is observed in the bandwidth which is reflected in the magnetoplasmon spectrum.

7. Conclusions

In conclusion, we have determined the inter- and intra-Landau-band magnetoplasmon spectrum for a magnetically modulated two-dimensional electron gas in the presence of a perpendicular magnetic field. Our results show that magnetic Weiss oscillations occur in intra-Landau-band magnetoplasmon spectrum. Their origin lies in the interplay of the two physical length scales of the system i.e. the modulation period, and cyclotron diameter at the Fermi level. When the strength of the magnetic modulation potential is equal to the electric one, the magnetic Weiss oscillations in the magnetoplasmon spectrum are out of phase and occur with a larger amplitude compared to the electric Weiss oscillations in

a 2DEG. These results also exhibit that the Weiss oscillations depend on the temperature much less than that of the SdH oscillations. We have determined the effects of both the electric and magnetic modulations on the magnetoplasmon spectrum of 2DEG. These oscillations are affected by the relative phase of the two modulations and positions of the extrema of the oscillations depend on the relative strength of the two modulations. We find complete suppression of Weiss oscillations for particular relative strength of the modulations when the modulations are out of phase.

Acknowledgments

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